Let's talk about applied theory.
Let's look into the singular cases of above described algorithm to project finding (finding of points, where dot product of tangent vector – in most cases it coincides with 1st derivative vector – and vector which joins the source point with the project point is equal to zero).
Dot product is equal to zero if at least one condition is observed:

1. Vectors are orthogonal.
2. Magnitude of at least one vector is equal to zero.

If only condition 1 is observed, found point sought for project point definitely.
Condition 2 is observed in one of two cases only:

2.1 Source point is on the curve (in this case source point is project point).
2.2 1st derivative of curve (in some point) is equal to zero. This point called "singularity point".

Case 2.2 requires more detail analysis.
In this case tangent vector may be computing successfully by numeric differentiation methods. It is made in new branch. It is not need detain us here. Only one question: Does it help us to find a project always?

In the figure 1 point O is singularity point, OA is tangent to curve in point O. Point $P \in OA$ and point $P_1 \in OA$.

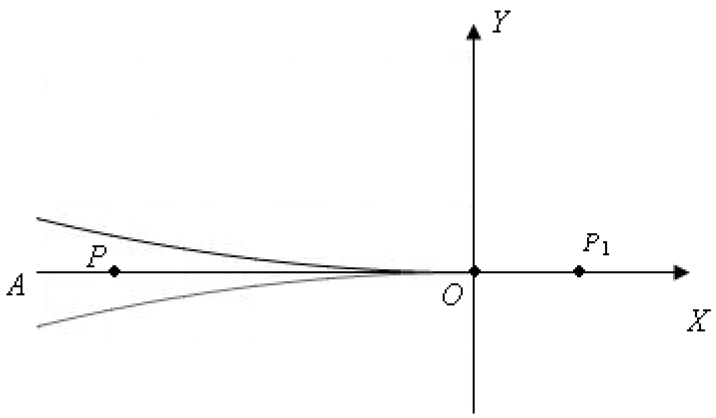

Figure 1

According to existing algorithm, point O is not project point of points P and P_1 because $PO \perp OA$ (the tangent to curve) and $P_1O \perp OA$.
On the other hand point O is extrema (point of local minimum or maximum of distance) for points P and P_1. So may seem O must be project point for P and P_1.
But let's look to problem more widely. Should we consider project point every point which is an extrema? If we answer "YES", then we should consider O is project point for all P_i points which are drawing in the figure 2 (and for any point of XY-plane generally). Is it acceptably?

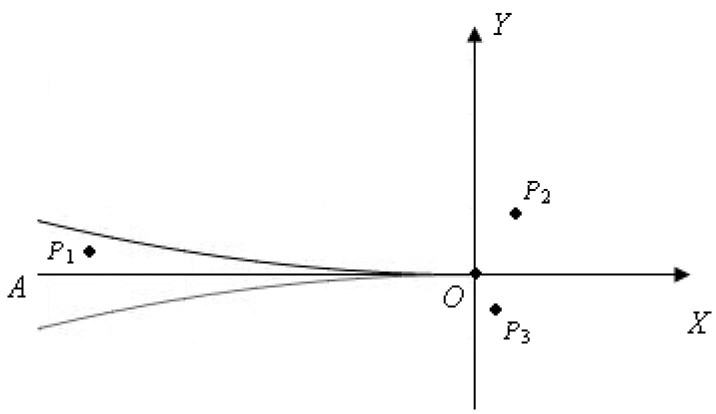

Figure 2

We can pose other criterion for project point: singularity point O is project for point P if OP is a tangent to curve in point O. But let's look to the figure 3 where shape looks like an infinity symbol. OP is a tangent to curve, but singularity point O is not even extrema for P. Should we consider O is a project point for P?

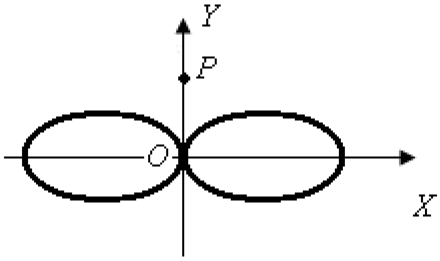

Figure 3

There are many questionable cases. So it is difficulty to determine all criterions of finding project point when there are singularity points. According to existing algorithm singularity point O is project point for point P if and only if at least one condition is observed:

a) OP is normal to tangent OA (for example case which is showed in the picture 4)
b) Points O and P are coincided.

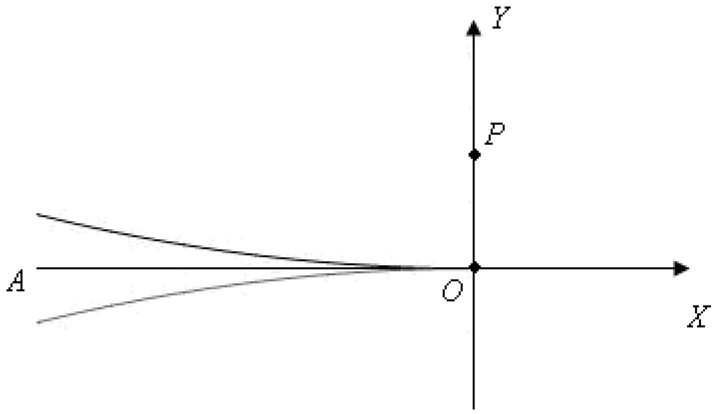

Figure 4

But there is one nuance when we use a) case. Cause of it is using of bisection method (and many other numeric methods of find roots).
Bisection method finds roots of equation "$f(x)=0$" successfully, if function $f(x)$ is continuous. In the case, which is showed in the figure 4, function $f(x)$ has appearance (approximately), which is showed in the figure 5. $x=a$ and $x=b$ are boundary of root finding (they match with singularity point not always), $x=c$ is root of equation (it corresponds to singularity point).

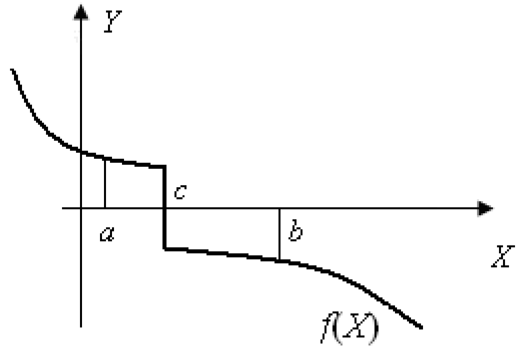

Figure 5

In this case, bisection method will find a root (always returns some value) $x=x_1$, but in most case x_1 won't be equal to c. However $|x_1 - c| < tol$ always. All the same we must be ready to get function value $f(x_1) >> 0$ or $f(x_1) << 0$.

To avoid it, we must to find all singularity points before solve of equation and exclude them from segments of root finding (root finding must be done in segments where function is continuous). But should we do it?