

Analytical intersection result.

Let consider an intersection of a plane with analytic representation

$$x + p = 0 \quad (1)$$

and a surface, which is result of revolution a line

$$\begin{cases} x_l = x_l(V) = a_x \cdot V \\ y_l = y_l(V) = b_y \\ z_l = z_l(V) = a_z \cdot V + b_z \end{cases}$$

around Z-axis.

In our case, we have following values of variables:

$$\begin{aligned} p = 25, \quad b_y = 25, \quad b_z = -25, \\ a_x = -0.447213595499958, \quad a_z = 0.894427190999916 \\ a_z / a_x = -2 \end{aligned} \quad (2)$$

Result of revolution is surface

$$\begin{cases} x_s = x_s(U, V) = \sqrt{x_l^2 + y_l^2} \cos U = \sqrt{a_x^2 V^2 + b_y^2} \cos U \\ y_s = y_s(U, V) = \sqrt{x_l^2 + y_l^2} \sin U = \sqrt{a_x^2 V^2 + b_y^2} \sin U \\ z_s = z_s(U, V) = a_z \cdot V + b_z \end{cases}$$

Let us note that

$$\begin{aligned} V &= \frac{z_s - b_z}{a_z}; \\ x_s^2 + y_s^2 &= a_x^2 V^2 + b_y^2 \\ x_s^2 + y_s^2 &= \frac{(z_s - b_z)^2}{\left(\frac{a_z}{a_x}\right)^2} + b_y^2 \\ \frac{x_s^2}{b_y^2} + \frac{y_s^2}{b_y^2} - \frac{(z_s - b_z)^2}{\left(\frac{b_y a_z}{a_x}\right)^2} &= 1. \end{aligned} \quad (3)$$

Line (3) is an one-sheet hyperboloid.

Let intersect hyperboloid (3) with plane (1). Intersection result satisfies an equation system

$$\begin{cases} \frac{x^2}{b_y^2} + \frac{y^2}{b_y^2} - \frac{(z - b_z)^2}{\left(\frac{b_y a_z}{a_x}\right)^2} = 1 \\ x + p = 0 \end{cases}$$

or, for our case (see (2)),

$$\begin{cases} \frac{x^2}{25^2} + \frac{y^2}{25^2} - \frac{(z+25)^2}{50^2} = 1 \\ x + 25 = 0 \end{cases}$$

From 2nd system equation follows $x = -25$. Let substitute it in 1st equation.

$$\begin{aligned} 1 + \frac{y^2}{25^2} - \frac{(z+25)^2}{50^2} &= 1 \\ \frac{y^2}{25^2} - \frac{(z+25)^2}{50^2} &= 0 \\ \pm \frac{y}{25} &= \frac{z+25}{50} \\ z &= \pm 2y - 25. \end{aligned} \tag{4}$$

I.e. as intersection result, we have two lines in given plane (1), which intersect in $(-25; 0; -25)$ point.